# Pricing Parisian and Par*asian* options analytically

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In this talk, two exact and analytic solutions for the valuation of European-style Parisian and Par*asian* options under the Black-Scholes framework are respectively presented. To the best of our knowledge, closed-form analytic formulae have never been found for the pricing of Parisian and Par*asian* options, although quite a few approximate solutions and numerical approaches have been proposed. A key feature of our solution procedure is the reduction of a three-dimensional problem to a two-dimensional problem through a coordinate transform that has elegantly ``absorbed" the directional derivative associated with the ``barrier time" into the time derivative and thus resulted in two coupled, but simplified PDE systems. For Parisian options, the coupled PDE systems are then analytically solved by applying the Laplace transform technique in conjunction with the construction of ``moving windows", which are introduced to evaluate the option prices backwards, slide by slide, until the value of the option at a given time and trigger value is found for a given underlying price. On the other hand, due to the non-resetting mechanism of the Par*asian* option, the coupled PDE systems of this type of options are much more complicated than those of their Parisian counterparts, and the ``moving window" technique fails in this case. Alternatively, the double Laplace transform technique is then applied to solve for the option prices in the Laplace space. However, our success of obtaining closed-form analytical solution hinges on overcoming the difficulty of performing Laplace inversions analytically. Finally, we have compared the results obtained from the newly-derived analytical solutions with those obtained through a numerical solution procedure. Such a comparison has not only reinforced the correctness of our newly-derived analytical solutions from numerical point of view, but also has demonstrated the efficiency of using the newly-derived analytical solutions to calculate option prices in finance practice.