Centre of Financial Mathematics Seminar

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Model Predictive Control for Pairs Trading Portfolio: Incorporation of Transaction Cost and Gross Exposure Constraints

Yuji Yamada[†], James A. Primbs[‡]

† University of Tsukuba, Tokyo, Japan

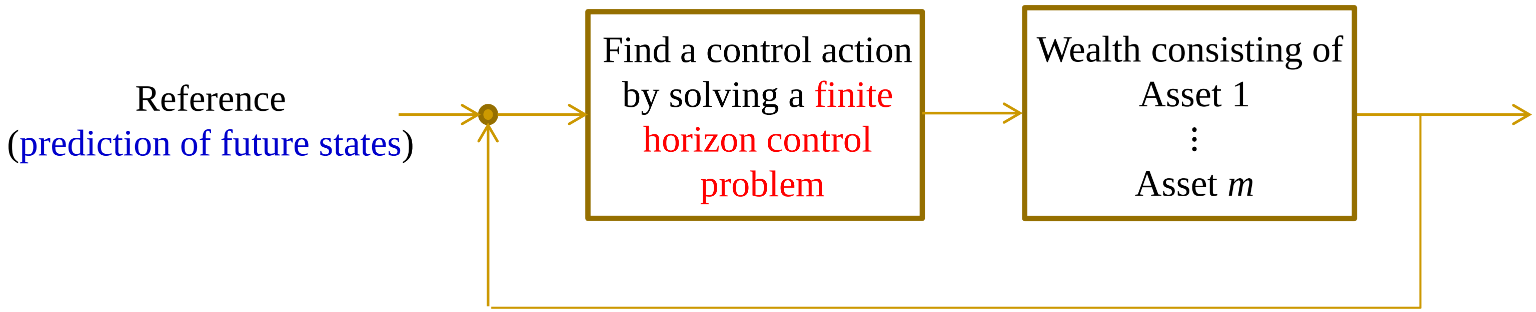
‡ Cal State University Fullerton, CA, USA

E-mail: yuji@gssm.otsuka.tsukuba.ac.jp

http://www2.gssm.otsuka.tsukuba.ac.jp/staff/yuji/

Model predictive control (MPC)

- A control methodology applied to portfolio optimization recently, in which a finite horizon control problem is solved based on the prediction of future state and output variables
- Only the initial control input is implemented and is updated online

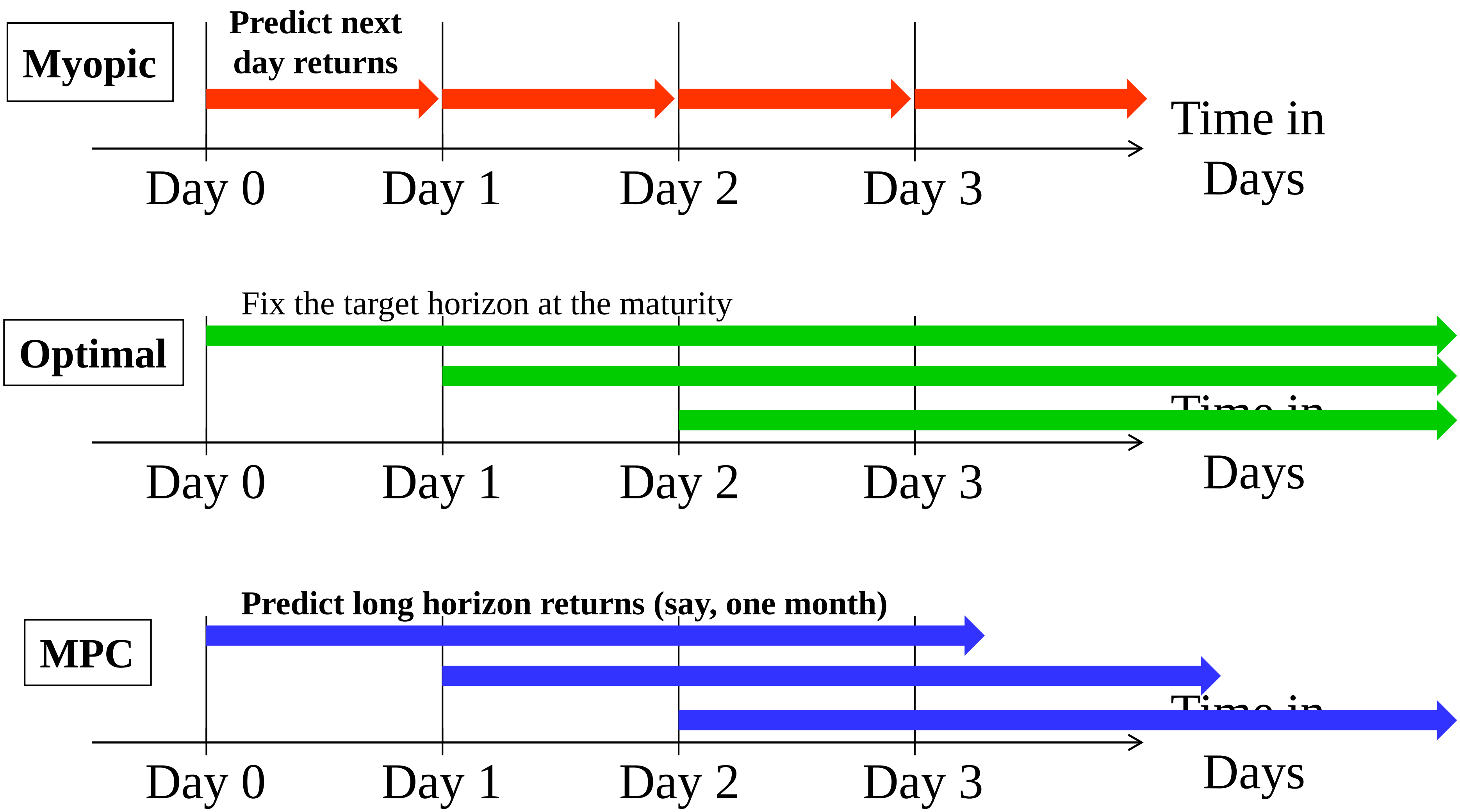


(e.g., Piccoli and Marigo (2004), Herzog (2005), Herzog et al. (2006, 2007), Meindl (2006), Primbs and Sung (2008), Sridharan et al. (2011), Dombrovskii et al. (2004, 2005, 2006), and so on)

⇔ Myopic portfolio: 1 day prediction horizon

⇔ Long term optimal portfolio: Dynamic yet fixed policy over the entire control horizon.

Myopic vs. MPC vs. Long term optimal



Model predictive control for pairs trading

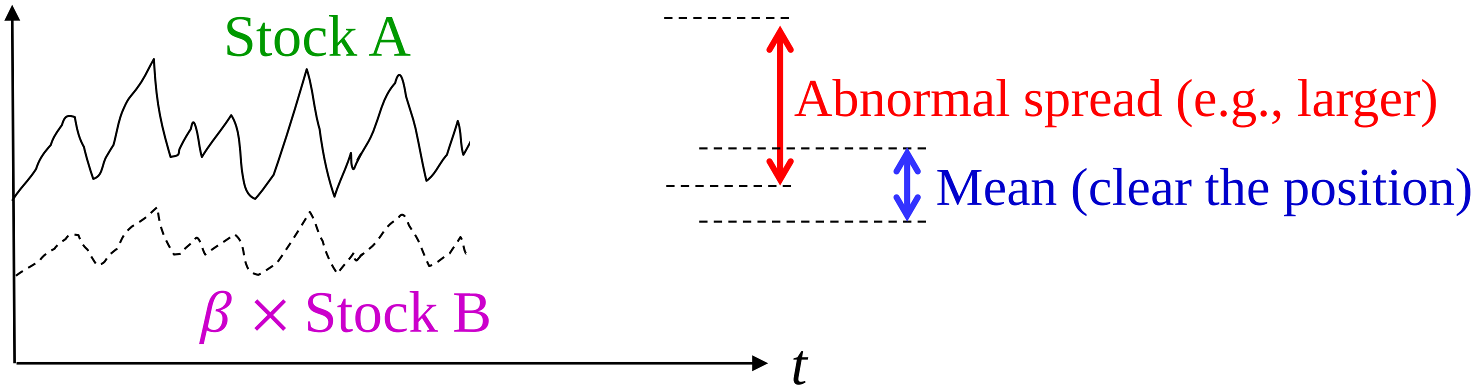
Extend our previous work in Yamada and Primbs (2012) to incorporate transaction cost and gross exposure constraints

Outline

- MPC approach for pairs trading portfolio optimization
- Transaction cost and gross exposure constraints
- Empirical simulation

Traditional pairs trading

Step 1) For a pair of stocks whose spread is mean-reverting, construct a long-short position when an abnormal spread is observed.



Spread = Price of Stock A – β × Price of Stock B

Step 2) Clear the position when the spread reverts towards its mean level in a future period, resulting in the profit of

|Abnormal spread – Mean spread|

(see Elliott et al. (2005), Gatev et al. (2006), Do et al. (2006), Mudchanatongsuk et al. (2008), Tourin and Yan (2013), Song and Zhang (2013), Deshpande and Barmish (2016), Yamamoto and Hibiki (2017) and references therein).

Pairs trading portfolio

- ✓ m pairs of stock prices, $\left(X_t^{(i)}, Y_t^{(i)}\right), i = 1, \ldots, m$ with spreads given as

$$S_t^{(i)} := X_t^{(i)} - \beta^{(i)} Y_t^{(i)}, \quad i = 1, \ldots, m$$

- ✓ Each spread provides a portfolio of a pair of stocks, and is invested as

$$u_t := \left[u_t^{(1)}, \ldots, u_t^{(m)}\right]^T : \text{Share unit vector on } S_t := \left[S_t^{(1)}, \ldots, S_t^{(m)}\right]^T$$

- The value of the wealth W_t with given prediction horizon $\tau \geq 1$:

$$W_{t+\tau} = u_t^T S_{t+\tau} + (1+r)^\tau \left(W_t - u_t^T S_t\right) = (1+r)^\tau W_t + u_t^T \left(S_{t+\tau} - (1+r)^\tau S_t\right)$$

- Total return $R_{t,\tau}$ between times t to $t+\tau$:

$$R_{t,\tau} := \frac{W_{t+\tau}}{W_t} = (1+r)^\tau + \frac{u_t^T}{W_t}\left[S_{t+\tau} - (1+r_f)^\tau S_t\right]$$

Construction of MPC

✓ MPC strategy may be formulated based on a conditional MV optimization problem for any given prediction horizon (Yamada and Primbs (2012))

$$\max_{u_t \in \Re^m} \left\{ \mathrm{E}_t\left[R_{t,\tau}\right] - \frac{\gamma}{2} \cdot \mathrm{V}_t\left[R_{t,\tau}\right] \right\}, \quad R_{t,\tau} := \frac{W_{t+\tau}}{W_t} = (1+r)^\tau + \frac{u_t^T}{W_t}\left[S_{t+\tau} - (1+r)^\tau S_t\right]$$

- Mean-reverting spread process:

$$VAR(q): S_t = \Phi_1 S_{t-1} + \cdots + \Phi_q S_{t-q} + c + e_t, \ \Phi_i \in \Re^{m \times m}, \ c \in \Re^m$$

$$S_t := \left[S_t^{(1)}, \ldots, S_t^{(m)}\right]^T, \quad S_t^{(i)} := X_t^{(i)} - \beta^{(i)} Y_t^{(i)}, \quad i = 1, \ldots, m$$

$$e_t \sim N(0, \Sigma), \quad \Sigma \in \Re^{m \times m}$$: covariance matrix

➡ A closed form solution is obtained in Yamada and Primbs (2012) as a function of observed spreads and the wealth.

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Consideration of transaction costs

- Proportional transaction cost incurred at time t:

$$\sum_{i=1}^{m} \rho \left(\left| X_t^{(i)} \right| + \left| \beta^{(i)} Y_t^{(i)} \right| \right) \left| \Delta u_t^{(i)} \right|, \qquad \Delta u_t^{(i)} := u_t^{(i)} - u_{t-}^{(i)}$$

Absolute value of the sum of positions in $X_t^{(i)}$ and $Y_t^{(i)}$

Changes in shares in the i-th spread

- Wealth & Total return

$$W_{t+\tau} = u_t^{\mathrm{T}} S_{t+\tau} + (1+r)^{\tau} \left(W_t - u_t^{\mathrm{T}} S_t - \sum_{i=1}^{m} \rho \Gamma_t^{(i)} \left| \Delta u_t^{\mathrm{T}} \right| \right), \quad \Gamma_t^{(i)} := \left| X_t^{(i)} \right| + \left| \beta^{(i)} Y_t^{(i)} \right|$$

$$R_{t,\tau}(\rho) = \underbrace{(1+r)^{\tau} + v_t \left[S_{t+\tau} + (1+r)^{\tau} S_t \right]}_{R_{t,\tau}(0)} - \underbrace{(1+r)^{\tau} \sum_{i=1}^{m} \rho \Gamma_t^{(i)} \left| v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \right|}_{\text{Transaction cost per wealth measured at time } t}, \quad v_t^{(i)} := \frac{u_t^{(i)}}{W_t}$$

Transaction cost (TC) constraint

- Introduce a set of new variables:

$$\kappa_t^{(i)} = \left| v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \right|, \quad R_{t,\tau} = R_{t,\tau}(0) - (1+r)^{\tau} \sum_{i=1}^{m} \rho \Gamma_t^{(i)} \kappa_t^{(i)}$$

$$\mathrm{E}_t[R_{t,\tau}] = \mathrm{E}_t[R_{t,\tau}(0)] - (1+r)^{\tau} \sum_{i=1}^{m} \rho \Gamma_t^{(i)} \kappa_t^{(i)}, \quad \mathrm{V}_t[R_{t,\tau}] = \mathrm{V}_t[R_{t,\tau}(0)]$$

- Conditional MV problem with TC constraint parameter ρ_c

$$\max_{v_t \in \Re^m} \; \mathrm{E}_t[R_{t,\tau}(0)] - \frac{\gamma}{2} \cdot \mathrm{V}_t[R_{t,\tau}(0)] - (1+r)^{\tau} \sum_{i=1}^{m} \rho_c \Gamma_t^{(i)} \kappa_t^{(i)}$$

$$\kappa_t^{(i)} \geq \left| v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \right| \Leftrightarrow -\kappa_t^{(i)} \leq v_t^{(i)} - \frac{u_{t-}^{(i)}}{W_t} \leq \kappa_t^{(i)}$$

⇒ Convex quadratic programing problem

Gross Exposure (GE) constraint

- In practice:
  - Percentage of the size of positions exposed to market risk

$$\text{GE} = \frac{|\text{Long positions}| + |\text{Short positions}|}{\text{Total wealth}}$$

  - If, for example, \$100 million has been invested in a fund, which has sold short \$50 million of stock and holds \$60 million worth of shares, gross exposure is \$110 million: 110 divided by 100 times 100 =110 percent.

- In academic literature:

  Constraint on the sum of absolute values of weights

  (Fan et al. (2012) and Qiu et al. (2015))

Gross Exposure (GE) constraint

- Absolute value of long and short positions in the i-th spread:

$$\left|u_t^{(i)}\right|\left(\left|X_t^{(i)}\right|+\left|\beta^{(i)}Y_t^{(i)}\right|\right)=\left|u_t^{(i)}\right|\Gamma_t^{(i)}$$

- GE constraint: $\sum_{i=1}^{m}\frac{\left|u_t^{(i)}\right|\Gamma_t^{(i)}}{W_t}=\sum_{i=1}^{m}\left|v_t^{(i)}\right|\Gamma_t^{(i)}\le\lambda$

$$\Leftrightarrow\quad\sum_{i=1}^{m}\xi_t^{(i)}\Gamma_t^{(i)}\le\lambda,\ -\xi_t^{(i)}\le v_t^{(i)}\le\xi_t^{(i)}$$

- Conditional MV problem with TC and GE constraints:

$$\max_{v_t\in\Re^m}\ \mathrm{E}_t\left[R_{t,\tau}(0)\right]-\frac{\gamma}{2}\cdot\mathrm{V}_t\left[R_{t,\tau}(0)\right]-(1+r)^{\tau}\sum_{i=1}^{m}\rho_c\Gamma_t^{(i)}\kappa_t^{(i)}$$

$$s.t.\ -\kappa_t^{(i)}\le v_t^{(i)}-\frac{u_{t-}^{(i)}}{W_t}\le\kappa_t^{(i)},\ \sum_{i=1}^{m}\xi_t^{(i)}\Gamma_t^{(i)}\le\lambda,\ \ -\xi_t^{(i)}\le v_t^{(i)}\le\xi_t^{(i)},\ i=1,\ldots,m$$

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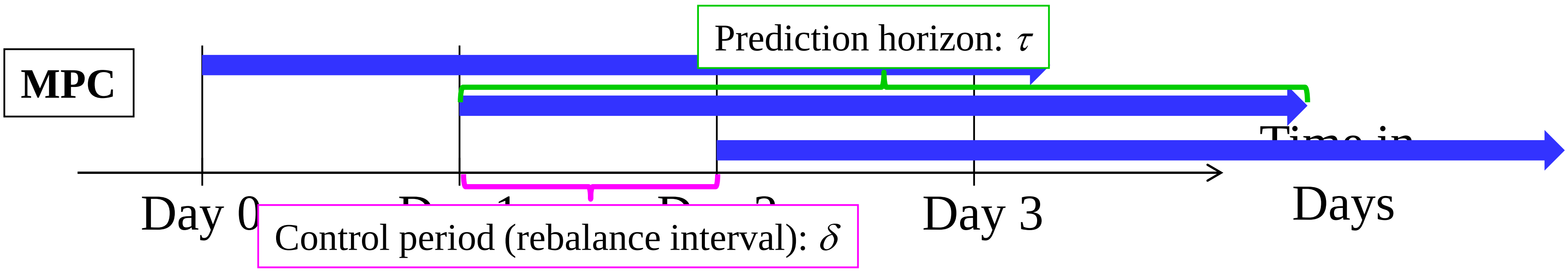
Empirical simulation

- Nikkei 225 as of Sep. 2016 with 5 years daily data (218 stocks)
- 3 year period: Oct. 2011 – Sep. 2014, Oct. 2012 – Sep. 2015,
  Oct. 2013 – Sep. 2016
  - Select pairs based on the pairs selection procedure in Yamada and Primbs (2012): 27 pairs
  - In-sample (2 year estimation period): Oct. 2013 – Sep. 2015
    ➡ Estimate required parameters including VAR coefficients
  - Out-of-sample (1 year simulation period): Oct. 2015 – Sep. 2016
    ➡ Apply the MPC algorithm by solving a QP problem

Empirical simulation

- Out-of-sample (1 year simulation period): Oct. 2015 – Sep. 2016

  ➡ Apply the MPC algorithm by solving a QP problem



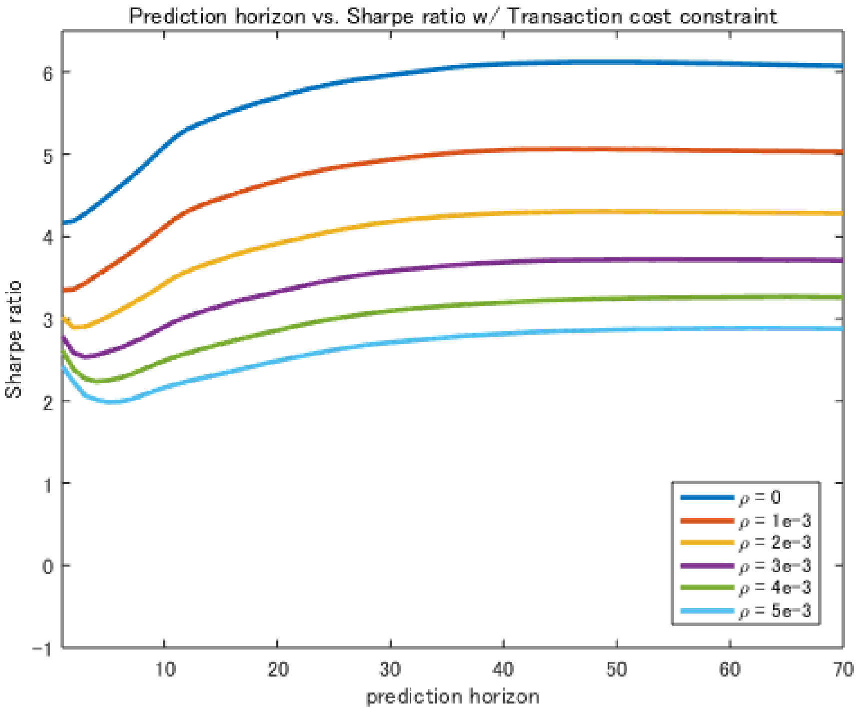
$$\text{Sharpe ratio} = \frac{\text{Mean rate of return} - \text{Risk free rate}}{\text{Standard deviation}} \quad \text{(Annualized)}$$

□ Parameter specification (if fixed or given):

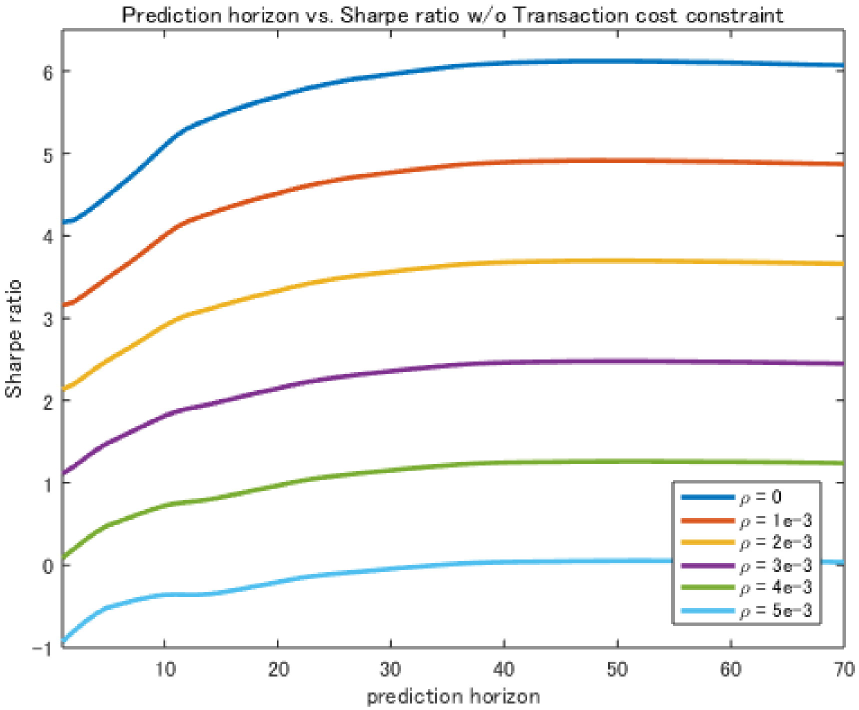
$$\gamma = 1 \times 10^3, \lambda = 1\ (GE), \rho_c = \rho\ (TC), \delta = 1\ (rebalance)$$

Prediction horizon vs. Sharpe ratio

w/ Transaction cost const.



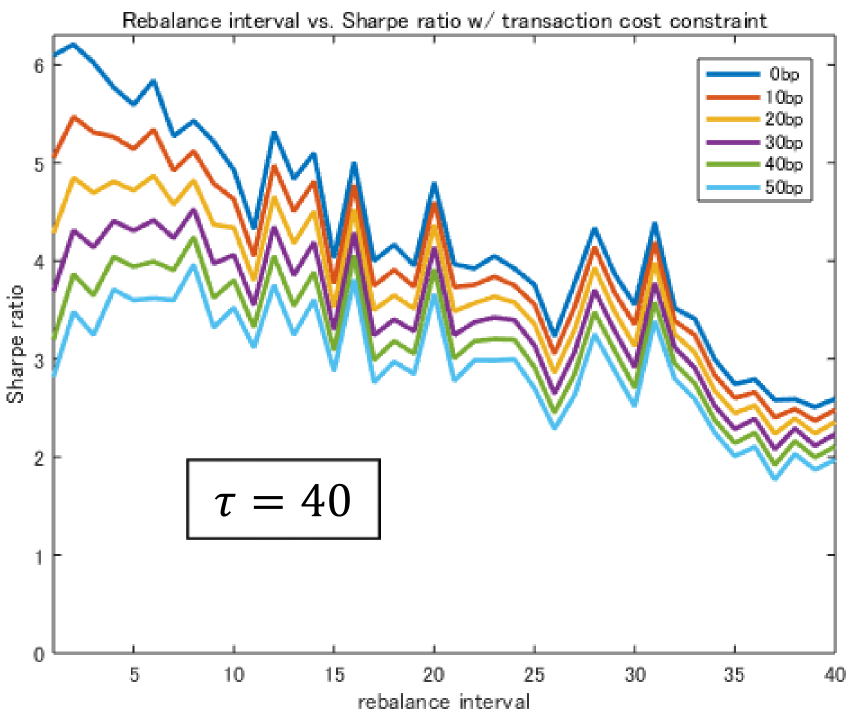
w/o Transaction cost const.



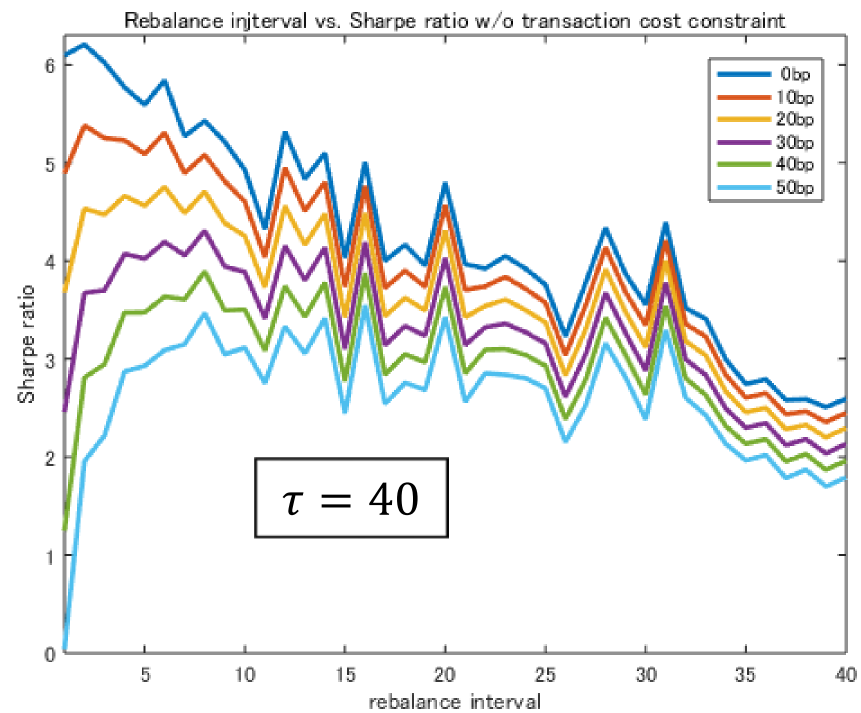
- The decrease in Sharpe ratio is slower if the TC constraint is incorporated.
- The Sharpe ratio tends to increase with the length of prediction horizon up to $\tau = 40 - 50$.

Rebalance interval vs. Sharpe ratio

w/ Transaction cost const.



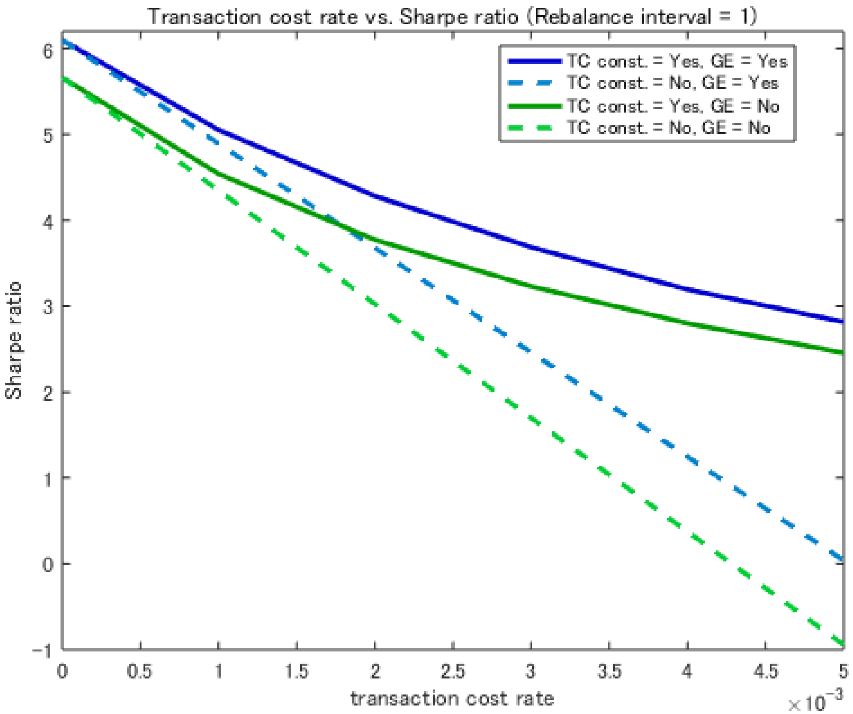
w/o Transaction cost const.



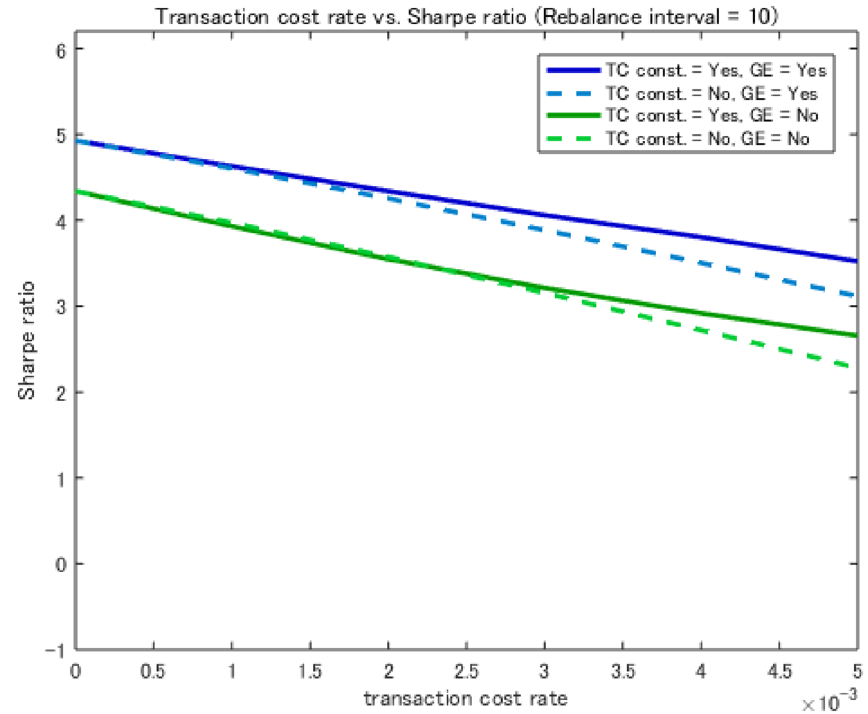
- The Sharpe ratio drops significantly as δ goes to 1 in the case of $\rho = 0.5\%$ for the MPC w/o TC constraint.
- Although the Sharpe ratio fluctuates as δ changes, and drops a little as δ moves closer to 1, there is not a clear dependence of the Sharpe ratio on δ for δ less than 20 for the MPC w/ TC constraint.

TC rate vs. Sharpe ratio w/ or w/o GE const.

Rebalance interval =1



Rebalance interval =10



- The Sharpe ratio is improved by incorporating the TC constraint ($\rho_c = \rho$).
- It can be further improved by adding the GE constraint.

Comparison with unconditional MV optimal portfolio

- Apply Markowitz model for $27 \times 2 = 54$ stock returns
  - Unconditional MV optimal portfolio: $\max_{w \in \Re^{2m}} \mathrm{E}[R_t] - \frac{\gamma}{2} \cdot \mathrm{V}[R_t]$

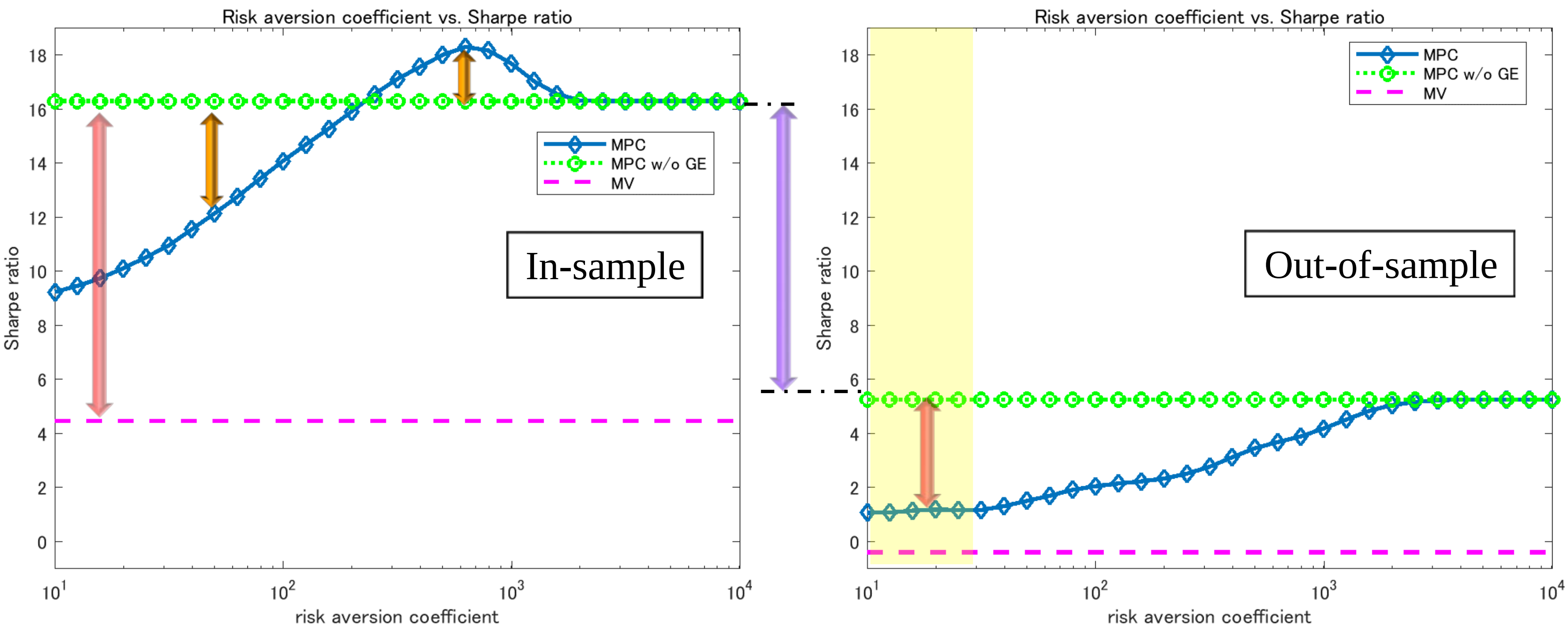
$$R_t = \sum_{j=1}^{2m} w_j R_t^{(j)}, \quad R_t^{(j)}: \text{Total return on stock } j = 1, \ldots, 2m$$

Effectiveness of using spread information and conditional MV approach vs. unconditional MV objective without spread information

- Estimate expected values and a covariance matrix of stock returns using the same parameter estimation period.
  - ✓ $\tau = \delta = 1$ in the MPC (which is myopic)
  - ✓ Vary γ ($= 10 \sim 10^4$)

MPC vs. MV optimal portfolio for different γ

- Parameter estimation period (In-sample): Oct. 2013 – Sep. 2015
- Simulation period (Out-of-sample): Oct. 2015 – Sep. 2016

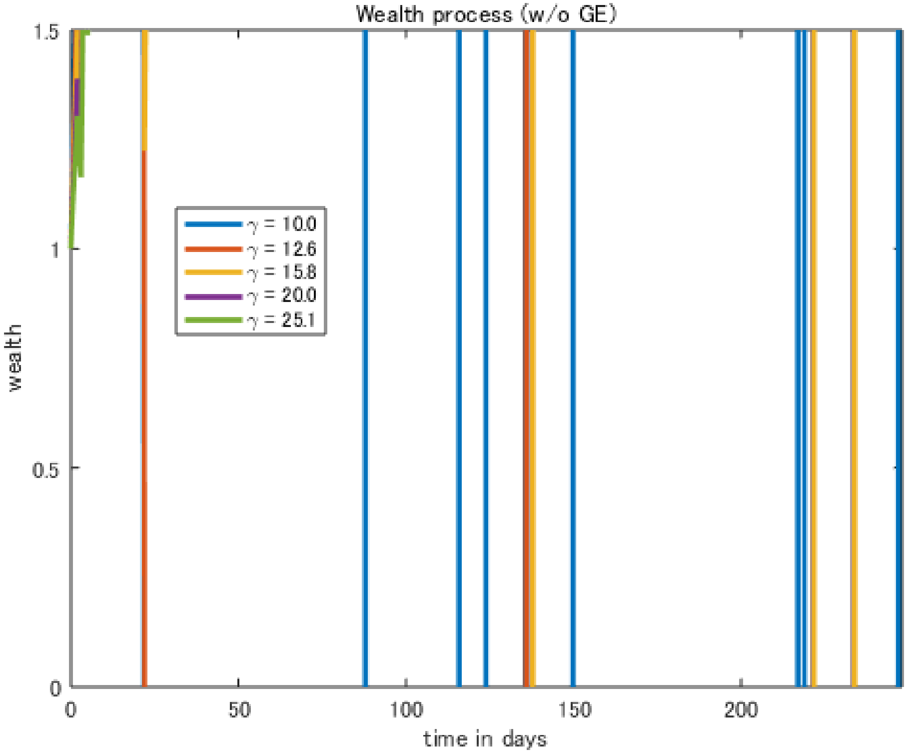


- The difference between the lines provides the effect of pairs vs. non-pairs, w/ GE vs. w/o GE, or in-sample vs. out-of-sample
- The GE constraint actually lowers the SR in myopic & out-of-sample cases

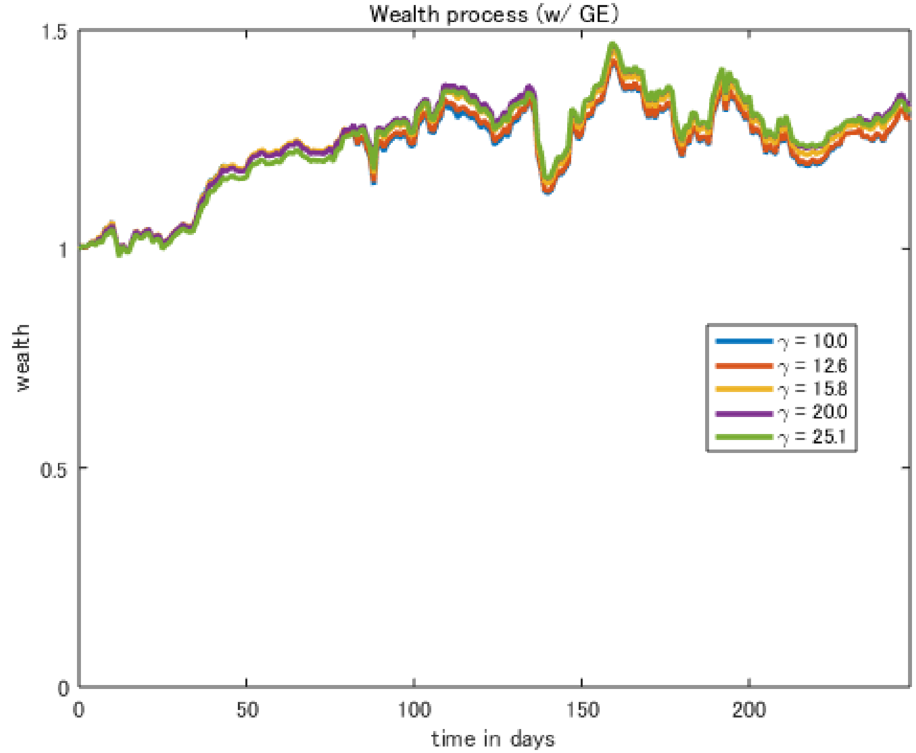
Role of GE constraint against default

➢ Selected sample paths of the wealth when $\gamma = 10, \dots, \gamma = 25.1$

MPC w/o GE constraint



MPC w/ GE constraint



- Cases that the wealth drops below zero, indicating that the fund is in default.
- The fund may be able to avoid the risk of default by imposing the GE constraint.

Summary

Model predictive control for pairs trading

Extended our previous work in Yamada and Primbs (2012) to incorporate transaction cost and gross exposure constraints

- ✓ MPC strategy for pairs trading portfolio is provided based on a conditional MV optimization problem.
- ✓ The conditional MV optimization problem is reduced to a convex quadratic problem even with the TC and the GE constraints.
- ✓ Incorporation of the transaction cost constraint improves the empirical performance of the wealth in terms of Sharpe ratio
- ✓ An important role of the GE constraint may be to avoid the risk of default